On the Impossibility of Stability-Based Equilibria in Infinite Horizon: An Example*

Alexander Frug† Malachy James Gavan‡

December 7, 2025

Abstract

This paper shows that stability-based equilibrium refinements may not be well defined when taken to the infinite horizon. To do so, we use a stable-set-style notion of the dynamically consistent partition, allowing for incomplete information. We provide a concrete example where, *only* via taking the game to the infinite horizon, the dynamically consistent partition of equilibria does not exist.

Keywords: Dynamically Consistent Partition, Revision-proof, Stability

*Frug acknowledges financial support from the Spanish Agencia Estatal de Investigación (AEI), through the Severo Ochoa Programme for Centres of Excellence in R&D (Barcelona School of Economics CEX2019-000915-S).

†Universitat Pompeu Fabra and Barcelona School of Economics; email: alexander.frug@upf.edu.

‡University of Liverpool; email: malachy.gavan@liverpool.ac.uk

1 Introduction

This paper shows that stability-based equilibrium refinements may not be well-defined in an infinite horizon setting. We demonstrate it in an environment where the refinement is well-defined in the finite horizon and an inconsistency arises only when extending to the infinite horizon. Specifically, we consider a concept of revision-proofness based on internal and external stability, in the spirit of Asheim (1997); Bernheim and Ray (1989) and Xue (2000), adapted to allow for incomplete information.[1] These works build on a logic similar to the notion of stability developed in Greenberg (1990)'s theory of social situations applied to equilibria of dynamic games.[2]

The general logic behind such solution concepts is to rule out plays that the players would abandon in favor of "robust" mutually preferred alternatives later on. A formal way to capture this is to define dynamically consistent partitions of equilibria—of the whole game and of its continuation games—into a "Good" set (robust to revision) and a"Bad" set (vulnerable to revision). Intuitively, an equilibrium belongs to the Good set if it only leads to continuation equilibria that also lie in the Good set, and at no point do players have an incentive to deviate to another such equilibrium. Conversely, an equilibrium is in the Bad set if, at some future point, the players will agree to switch to a better plan from the Good set. The requirement of dynamic consistency ensures that this classification holds throughout the game.

Existence of these concepts has been shown in specific settings, for instance finite horizon environments, infinitely repeated games, and infinite games with dynasties of players (see for example, Asheim (1997); Bernheim and Ray (1989); Xue (2000); Ales and Sleet (2014)). Our contribution is to show, through a concrete example, that a dynamically consistent partition may fail to exist in an infinite-horizon setting with incomplete information. While the method yields a natural classification of equilibria into revision-proof and non-revision-proof in every finite truncation of our game, this classification breaks down in a natural infinite extension. Specifically, it becomes impossible to identify any equilibrium as either revision-proof or not. This result serves

[1]For some recent contributions to adapting cooperative-based solution concepts to incomplete information, see Dutta and Vohra (2005); de Clippel (2007); Bloch and Dutta (2009); Salamanca (2020).

[2]Both Asheim (1997) and Xue (2000) point directly to this connection. Asheim (1997)'s main focus is on consistent planning problems of Strotz (1955) and Pollak (1968) while Xue (2000) looks at dynamic games. Bernheim and Ray (1989) study repeated games.

as a warning against applying such refinements in infinite-horizon environments.

We develop this result within a dynamic cheap-talk framework in which the sender undergoes exogenous gradual learning. As in standard cheap-talk models beginning with Crawford and Sobel (1982), the sender transmits non-verifiable messages about the state of the world, and the receiver takes an action based on those messages. In our setting, however, the sender acquires information gradually over time, according to a fixed exogenous process, and communicates repeatedly with the receiver.[3]

To further illustrate the environment and the logic of the solution concept that will be introduced shortly, we now turn to an example.

Example 1. *Consider the canonical uniform-quadratic case with the sender's constant bias $b = \frac{1}{8}$. That is, the sender and receiver's utility functions are, respectively,*

$$u_S(a, \theta) = -(\theta + \frac{1}{8} - a)^2 \text{ and } u_R(a, \theta) = -(\theta - a)^2,$$

where a is the action chosen by the receiver and $\theta \in [0, 1]$ is the uniformly distributed state of the world.

Now consider the learning process where, the sender is initially uninformed, then, in period 1, the sender learns whether the state is above or below 0.4, and finally, in period 2, the sender perfectly learns the state. Formally, denoting by L_t the sender's information partition in period t, we have

$$L_0 = \{[0, 1]\}, L_1 = \{[0, 0.4), [0.4, 1]\}, \text{ and } L_2 = \{\{\theta\} | \theta \in [0, 1]\}.$$

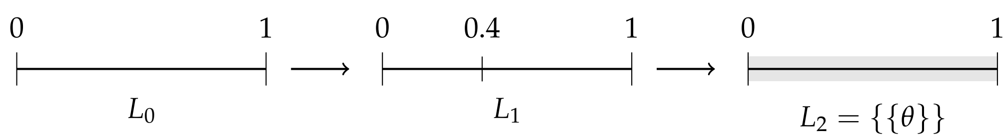

Figure 1: Learning process in Example 1.

If there is no informative communication in period 1, the only receiver's information partitions (up to interval boundaries) that are consistent with equilibrium (in period 2) are babbling

[3] Cheap talk with exogenous gradual learning was previously studied by Frug (2022). Similar environments but with endogenous learning processes were studied by Ivanov (2015, 2016) and Frug (2016, 2018).

and interval partition at the threshold 0.25*:*

$$\{[0,1]\}\ and \{[0,0.25),[0.25,1]\}.$$

However, the most informative equilibrium (that is preferred by both players ex ante) is simply to fully reveal the sender's information in period 1, inducing the receiver's information partition:

$$\{[0,0.4),[0.4,1]\}.$$

Let us consider the stability of coordinating on the efficient equilibrium. Notice that this equilibrium requires informative communication in period 1 and relies on no information being transmitted in period 2. Now, suppose that the communication in period 1 revealed that $\theta \geq 0.4$*. Then, at the beginning of period 2 (and, importantly, before the sender learns the state), the players are facing a continuation game where they share the same information – that the state is uniformly distributed on* $[0.4,1]$ *– and understand that the sender is about to learn the state perfectly. They also understand that the two (Crawford-Sobel) equilibria of this continuation game are babbling (which reveals no information) and informative which leads to the receiver's information partition*

$$\{[0.4,0.45),[0.45,1]\}.$$

Both players (the uninformed receiver and "all sender types" - in this case - a single information set, since the sender is uninformed too!) strictly prefer the informative equilibrium of the continuation game.

In analysing static communication games à la Crawford and Sobel (1982), it is often assumed that the players will seek to avoid dominated equilibria. By the same token, if the players are sufficiently sophisticated to coordinate on a Pareto efficient equilibrium ex-ante, why would they ignore opportunities to reach Pareto improvements down the road? In other words, why not revise at the beginning of the continuation game (which in this case, is identical to a Crawford and Sobel (1982) single-stage communication game)? The problem is that if the players expect to revise their plan and coordinate on the most informative equilibrium of the continuation game, it changes the incentives for truth-telling in period 1. Specifically, the sender who learns that the state is in $[0,0.4)$ *will prefer to report that the state is in* $[0.4,1]$*, planning to (mis)report that it belongs to* $[0.4,0.45)$ *in stage 2.*

Therefore, informative communication in stage 1 leads to a desire to revise in the future and *we need a solution concept that takes this into account since it seems unlikely that the players will follow a dominated equilibrium of the (continuation-) game just because they promised to do so in the past.* □

To more concretely pin down the idea of joint dynamic revision, we use a solution concept of a *dynamically consistent partition* (DCP), similar to the concepts of Asheim (1997) and Xue (2000), adapted to incomplete information. The concept we develop works as follows. We begin by considering all *continuation games*[4] and identifying the set of equilibria of each such game and the (stochastic) transition from a given game into its continuation games.[5] Then, for each such game, we partition the set of equilibria into those that are "revision-proof" and those that are not. Crucially, the resulting partition function (that assigns to each game a set of revision-proof equilibria of that game) must be dynamically consistent in the sense that a revision-proof equilibrium of a given game leads to a revision-proof equilibrium in any reachable continuation game (internal stability), and any non-revision-proof equilibrium of a given game leads (with positive probability) to a continuation game where the players will find a Pareto improving revision-proof equilibrium (external stability). We refer to such a partition function as *Dynamically Consistent Partition*, *DCP*, and to the set of revision-proof equilibria of the original game simply as *revision-proof* equilibria.

The behavioral principle we aim to capture is simple: At any point in time, the players seek to coordinate on a Pareto efficient equilibrium and, whenever possible, they are happy to replace the (continuation of the) equilibrium they currently plan to play in favor of a Pareto-improving equilibrium of the continuation game (they are currently at); however, they are willing to do so only if they expect that the new equilibrium will not be replaced later in favor of another Pareto-improving equilibrium (of some reachable continuation game). This notion uses the logic of Greenberg (1990)'s theory of social situations to refine the set of equilibria.

First, we illustrate the solution concept in the finite horizon. For example, in the single-period information transmission game of Crawford and Sobel (1982), the solution concept selects all the Pareto efficient equilibria. Note that this set depends on the

[4] This includes the original game and all the games that begin after a finite non-terminal sequence of reports.

[5] We note that this transition is determined not only by the sender's learning process but also by the equilibrium played (e.g., the receiver's information in any continuation game under the babbling equilibrium of the original game is that the state is uniform between zero and one).

assumption of the sender's initial information. Consider the baseline Crawford and Sobel (1982)'s variant of Example 1 – i.e., same preferences but suppose the sender is fully informed from the outset. In this case, any equilibrium is revision-proof since any equilibrium is top-ranked by some sender types. On the other hand, if the players begin the interaction before the sender knows the state, only the (most) informative equilibrium is selected. This is true in general for the uniform-quadratic specification since the players rank identically all receiver's information partitions from the ex-ante perspective. To illustrate the dynamic implications of revision-proofness in our context, we now turn back to Example 1 (including the sender's learning process).

Example 1 (Revisited)**.** *In the unique revision-proof equilibrium, information is transmitted only in period 2, when the sender reveals whether the state is above or below* 0.25*.*

The previously established logic rules out informative communication in period 1 (since it will lead to a continuation game where it will be revised).[6]

Now consider the communication in period 2 if the equilibrium played reveals no information in period 1. Since no information has been revealed yet, and the communication in period 2 takes place when the sender is perfectly informed, the equilibria of that continuation game are identical to Crawford-Sobel equilibria. That is, there is the babbling equilibrium that reveals no information and the informative equilibrium in which the receiver learns whether the state is below or above 0.25*. The informative equilibrium is a priori better than the babbling by the receiver and all sender types prior to learning* L_2 *(i.e., the sender "period 1 type" whose belief is that the state is distributed uniformly between* 0 *and* 0.4 *and the one who believes the state is distributed uniformly between* 0.4 *and* 1*). Therefore, the informative equilibrium is revision-proof in the continuation game of period 2 that begins after no information is revealed in period 1.*

Working backwards, consider the equilibrium of the entire game that begins with babbling in period 1 and continues with the revision-proof equilibrium described above for the continuation game of period 2. Refer to this equilibrium as "delayed-communication equilibrium."

Since the delayed-communication equilibrium continues with a revision-proof equilibrium, the players will not revise it in period 2. Do they want to revise it in period 1? Even though, prior to any sender's learning, both players prefer the most informative equilibrium (where the receiver learns whether the state is above or below 0.4), the players will not replace the delayed-communication equilibrium in favor of the most-informative equilibrium of the entire

[6]Note that mixed-strategy reporting is incompatible with equilibrium in this setting.

game since, as explained earlier, they understand that they may have an incentive to revise it in the future. Hence, the delayed-communication equilibrium is a revision-proof equilibrium of the entire game and it is unique since there are no other equilibria. □

In the above example, the solution concept led to a delay in communication. This arguably natural result is typical since, as time goes by, revising the course of play becomes more difficult since agreeing on Pareto improvements with a sender whose information is more refined requires more demanding constraints.[7]

The logic of partitioning the set of equilibria of the continuation games into revision-proof and non-revision-proof equilibria in a dynamically consistent manner satisfies some natural benchmarks for being well behaved. Specifically, whenever such a dynamically consistent partition exists, the set of revision-proof equilibria *of the entire game* is non-empty.[8] Hence, the result of the paper is not merely a result of the non-existence of revision-proof equilibria but a deeper negative result on the existence of dynamically consistent partitions.[9] In the example we provide, this result is driven *only* via taking the solution concept to the infinite horizon, as the solution concept is always non-empty in this setting with a finite horizon. Therefore, in our setting, it is only impossible to distinguish a revision-proof equilibrium from one that is not within the infinite horizon.

We show, using a concrete example, that when the time horizon is infinite the dynamically consistent partition may not exist. While it was suspected in the literature that the logic of such solution concepts does not easily extend to the infinite horizon (see, for example, Asheim (1997)), to the best of our knowledge, our example where the solution concept leads to non-existence in the infinite horizon while always exists in a finite horizon is the first in the literature. Therefore we provide a warning that some caution should be taken when applying such concepts to infinite horizon settings.

Before turning to the model and analysis, we first briefly discuss the literature.

[7]Qualitatively, this means that, in "later" continuation games—when the sender's information improves—fewer equilibria will fail to be revision-proof (e.g., in a typical single-stage communication game where the sender is perfectly informed from the outset, all equilibria are revision-proof since each such equilibrium is top-ranked from the perspective of some sender type).

[8]Note this is not the case in general when considering only the set of equilibria, see the literature review for further discussion.

[9]Under Asheim (1997)'s concept, there is no possibility of emptiness. However, when considering only the set of equilibria emptiness can occur. In the working paper version Asheim (1991) also connects his concept to where only equilibria are compared.

2 Literature Review

Several papers have introduced versions of revision-proofness in the literature (Bernheim and Ray, 1989; Farrell and Maskin, 1989; Asheim, 1997). Our notion of revision-proofness (or consistent partition) builds on the concept of Asheim (1997), and adapts it to incomplete information by allowing each information type to be treated as a player. Asheim (1997) relates his concept to coalition proofness of Bernheim et al. (1987) and the concept of consistent planning of Strotz (1955) and Pollak (1968), as well as other notions of revision-proofness such as Bernheim and Ray (1989); Farrell and Maskin (1989). Asheim (1997) uses the notion of optimistic stability, building on the work of Greenberg (1990)'s social situations. In addition, our notion also extends Xue (2000)'s "weakly negotiation-proof Nash equilibrium" which is shown to exist in finite games of complete information.

It is well known that there may be no renegotiation-proof plan (Hellwig and Leininger, 1987; Asheim, 1997).[10] This can occur even in finite horizon. The logic is as follows. Suppose that there is a *single* equilibrium of the original (multistage) game, where there are multiple continuation equilibria (but, due to uniqueness, only one such equilibrium is consistent with the equilibrium of the whole game). However, it is possible that the continuation equilibrium consistent with the unique equilibrium of the whole game is not renegotiation-proof (in stage 2). Rather upon arriving at that continuation, another equilibrium is preferred by all active players. In that case, it can disqualify the original equilibrium and therefore there is no revision-proof equilibrium.[11] However, this leads to the conclusion that there is no revision-proof equilibria. In contrast, our result will show that in some environments it is not possible to say whether or not *any* given equilibrium is revision-proof, as no dynamically consistent partition will exist.

When extending concepts of dynamic consistency to the infinite horizon, it is unclear what challenges can occur. Indeed, little is understood about this notion beyond the finite action finite horizon case. For example, in the discussion of optimistic stable

[10] It is worth pointing out that when considering all paths, as Asheim does, the dynamically consistent set may not be well defined. Relying on an inconsistency between internal and external stability. Such inconsistency does not occur when only defined over the set of equilibria, but rather emptiness of the set. See Asheim (1991) for further discussion.

[11] See example 6 of the working paper of Asheim (1997): Asheim (1991) for further discussion of this.

standards of behaviour (or a dynamically consistent partition) in a consistent planning environment with complete information, Asheim (1997) points out that "*with finite action but infinite horizon, no counterexample to existence is available*."

A related but distinct concept is that of the stable set. Both the stable set and the notion of a dynamically consistent partition in this paper can be seen as a special case of Greenberg (1990)'s stability in social situations. It is well known that the stable set may not exist (Lucas, 1969) – that is, it may not be possible to partition the set of outcomes into those that are stable and unstable, due to 'odd cycles' on the induced graph (Richardson, 1953). We show that the dynamically consistent partition may not exist, and the logic behind our construction is different from that used in Richardson (1953); Lucas (1969).

3 Definitions and preliminary analysis

Our starting point is the uniform quadratic specification of Crawford and Sobel (1982). That is, we assume that $\theta \sim U[0,1]$ and that the players' payoffs are given by:

$$u_S(a,\theta) = -(\theta + b - a)^2 \text{ and } u_R(a,\theta) = -(\theta - a)^2,$$

where a is the action chosen by the receiver and $b > 0$ is the sender's bias.

We now introduce several preliminary definitions that will be needed later to define the notion of revision-proofness in terms of dynamically consistent partitions. While the definitions make use of some simplifying specification-related assumptions (such as the uniformly distributed state of the world), we prefer to offer a sufficiently general form of the definitions (that may go slightly beyond what is strictly necessary for our example) to fully illustrate the logic of the concept.

A *reporting game* $G(I^R, L)$ consists of the receiver's initial information—a (measurable) set $I^R \subset [0,1]$ on which the receiver believes that the state is uniformly distributed,[12] and the sender's learning process $L = \{L_t\}_{t=0}^T$ on I^R where T is finite or infinite, L_t is a partition of I^R such that for each $t < T$, L_{t+1} is a refinement of L_t (i.e.,

[12]The uniform distribution of the state for each continuation game in fact follows from our assumption that the prior distribution of the state is uniform (as we build on the uniform quadratic specification of Crawford and Sobel (1982)), and the sender's reports will form partitions over time.

L_t can be obtained from unions of elements of L_{t+1}), and so the set L_0 represents the sender's initial information partition over I^R.

Given a reporting game G, denote by $EQ(G)$ the set of Bayesian Nash equilibria of G.[13] Note that the set of equilibria is always non-empty as babbling is always consistent with an equilibrium. In addition, for every G, let $L_{0(G)}$ denote the sender's information partition at the beginning of G (which we denoted L_0 when the game was fixed).

Comparing Equilibria. For $e_1, e_2 \in EQ(G)$, we say that e_1 *Pareto dominates* e_2 (denote $e_1 \succ e_2$) if e_1 is preferred over e_2 by the receiver and *all* sender's $L_{G(0)}$ elements. Some of the preferences may be weak but at least one strict preference is required.

Reachable Reporting Games. Denote by G^0 the original game (that marks the beginning of the interaction) and let $L = \{L_t\}_{t=0}^{T}$ be the sender's learning process of $I^R = [0, 1]$ in G^0. Given $t \leq T$, denote by $\sigma(L_t)$ the set of all non-empty (measurable) unions of elements of L_t. Denote by:

$$\Gamma_t(L) = \left\{ G(I^R, \hat{L}_t) \,\middle|\, I^R \in \sigma(L_t), \hat{L}_t \text{ is the continuation of } L \text{ from period } t \text{ restricted to } I^R \right\}$$

the set of reporting games that can be reached at period t, and denoted by:

$$\Gamma(L) = \bigcup_{t \leq T} \Gamma_t(L)$$

the family of all $L -$ *reachable* reporting games.

For $G_1, G_2 \in \Gamma(L)$ and $e \in EQ(G_1)$, let us write:

$$G_1 \underset{e}{\rightarrow} G_2$$

if the game G_2 is obtained with positive probability according to the reporting strategy played under the equilibrium e of the game G_1.

Revision-Proof Equilibria. Next, for all $G \in \Gamma(L)$, we attempt to provide a consistent

[13] Formally, our definition only allows for pure reporting strategies. The results would not change with a more general definition. However, this would come at the cost of heavier notation and therefore is omitted.

partition of the sets $EQ(G)$ into equilibria that are revision-proof and those that are not. This dynamic consistency is formalized as follows:

A dynamically consistent partition (DCP) with respect to $\succ$, is a collection of subsets $RP(G) \subset EQ(G)$, for all $G \in \Gamma(L)$ that satisfies:

- *Internal Stability* For each $e \in RP(G)$, for each $\tilde{G}$ with $G \underset{e}{\rightarrow} \tilde{G}$ and for each $\tilde{e} \in RP(\tilde{G})$, $\tilde{e} \not\succ e|_{\tilde{G}}$.
- *External Stability* For each $e \notin RP(G)$, there is $\tilde{G}$ with $G \underset{e}{\rightarrow} \tilde{G}$ and there is $\tilde{e} \in RP(\tilde{G})$ with $\tilde{e} \succ e|_{\tilde{G}}$.

We note that in this setting, the solution concept is well-behaved for finite horizon. That is, it is non-empty valued and well-defined.[14]

In general, if a dynamically consistent partition exists, in this setting, a revision-proof equilibrium of the entire game exists. We note that this is not generally the case, and there exist many settings for which revision-proof equilibria do not exist, as discussed within the literature review.

Proposition 1. *If DCP exists, a revision proof equilibrium of the entire game G^0 exists.*

We note that a similar result exist Asheim (1997) (proposition 2.1). However, as our solution concept is for incomplete information and only compares equilibria (rather than all paths, as Asheim's concept does), we cannot use that result directly.

Proof. Recall that $EQ(G^0) \neq \emptyset$ as $B \in EQ(G^0)$ where B denotes the completely uninformative (babbling) equilibrium. Assume by contradiction that $RP(G^0) = \emptyset$. In particular $B \notin RP(G^0)$, so there is a minimal $\tau > 0$ with $\tilde{G} \in \Gamma_\tau(L)$ such that $G^0 \underset{B}{\rightarrow} \tilde{G}$ where $\tilde{G} = G([0,1], \hat{L}_\tau)$, $\hat{L}$ is a $\tau-$continuation of L, and there is $\tilde{e} \in RP(\tilde{G})$ with $\tilde{e} \succ B|_{\tilde{G}}$.

Notice that if babbling is played before period τ, the continuation game $\tilde{G}$ is reached with certainty. Consider the reporting strategy that is composed of babbling until time τ, and the reporting strategy as in $\tilde{e} \in EQ(\tilde{G})$. It is immediate that this reporting strategy is consistent with equilibrium. Denote it by $e \in EQ(G^0)$. By assumption, $e \notin RP(G^0)$.

[14]Interested readers can request the formal statements and proofs of these results from the authors.

Since $\tilde{e} \in RP(\tilde{G})$, there does not exist $s \geq \tau$, $G' \in \Gamma_s(L)$ and $e' \in RP(G')$ such that $G^0 \underset{e}{\to} G'$ and $e' \succ e|_{G'}$. Then, there is $s < \tau$, $G' \in \Gamma_s(L)$ and $e' \in RP(G')$ such that $G^0 \underset{e}{\to} G'$ and $e' \succ e|_{G'}$. Recall that all elements of L_τ agree that $\tilde{e}$ is at least as good as transmitting no information to the receiver. Since L_s is a coarsening of L_t, it is also true that all elements of L_s agree that $\tilde{e}$ is at least as good as transmitting no information to the receiver. Thus, all elements of L_s agree that e' is at least as good as transmitting no information to the receiver. Since e' is informative, the receiver prefers e' over the completely uninformative equilibrium as well. To conclude, notice that $G^0 \underset{B}{\to} G'$ and therefore we get a contradiction to the definition of τ because $e' \succ B|_{G'}$. $\square$

4 Infinite Horizon

The definitions in section 3 (and Proposition 1) allow for finite as well as infinite horizon. In this section, we provide a concrete counterexample to the existence of DCP for infinite horizon. While it was pointed out that the existence of such infinite horizon families of partitions (under related concepts) is not guaranteed (e.g., see the discussions in and Asheim (1997))[15], providing counterexamples proved to be challenging and, to the best of our knowledge, concrete counterexamples did not appear in the literature.

Example 2. *Consider the uniform-quadratic case with the sender's constant bias $b = \frac{1}{12}$. That is, $\theta \sim U[0,1]$ and the sender and receiver's utility functions are, respectively,*

$$u_S(a,\theta) = -(\theta + \frac{1}{12} - a)^2 \text{ and } u_R(a,\theta) = -(\theta - a)^2.$$

Consider the increasing sequence $\{x_j\}_{j=0}^{\infty}$, where $x_j = \frac{1}{3} \cdot \frac{j}{j+1}$, and suppose the sender's learning process is given by

$$L_t = \{\{\theta | \theta \in [0, x_t)\}, [x_t, 1]\}.$$

This means that the sender is initially uninformed but if the state is below x_t (and only then), the sender will learn the state perfectly within the first t periods, and since $x_t \to \frac{1}{3}$, states (weakly)

[15] Note that in the related concepts of Bernheim and Ray (1989) and Farrell and Maskin (1989) existence of a revision-proof equilibrium (leading to the empty set of revision-proof equilibrium) and both discuss the difficulty of extending such concepts to the infinite horizon.

above $\frac{1}{3}$ will always belong to the (unique) non-degenerate interval for each t.

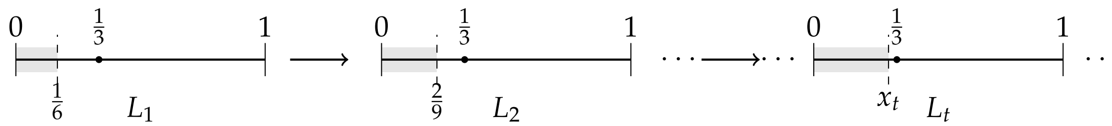

Figure 2: Learning process in Example 2.

It is easy to see that the set of equilibria of this game is given by

$$EQ(G^0) = \{e_j | j \in \{0\} \cup \mathbb{N}\},$$

*where e_j represents the equilibrium in which the sender reports at $t = j$ either "*low*" or "*high*" report, where* low *means $\theta < x_j$ and* high *means $\theta \geq x_j$ and in all other periods, the reports are uninformative (babbling). Note that e_0 represents the babbling equilibrium (which we denoted B). A convenient feature of this example is the comparison of equilibria. Specifically, note that*

$$e_{j_2} \succ e_{j_1} \iff j_2 > j_1.$$

Imagine that e_k is played for some $k \in \mathbb{N}$. After the report at $t = k$, two games can be induced, G^k_{low} and G^k_{high} after the "low" and "high" reports respectively. Note that $EQ(G^k_{low}) = \{B\}$, that is, the only equilibrium of the continuation game is the completely uninformative equilibrium, and $EQ(G^k_{high}) = \{e^k_j | j \geq k, j \in \mathbb{N}\}$, where e^k_k represents the completely uninformative equilibrium of the continuation game, and

$$e^k_{j_2} \succ e^k_{j_1} \iff j_2 > j_1.$$

Claim 1. *A dynamically consistent partition does not exist.*

Proof. Assume by way of contradiction that a *dynamic consistent partition* exists. Then, by Proposition 1 and the fact that Pareto ranking is complete, there exists $j \in \{0\} \cup \mathbb{N}$ such that $RP(G^0) = \{e_j\}$. In particular, $e_{j+1} \notin RP(G^0)$. It follows that $e^{j+1}_{j+1} \notin RP(G^{j+1}_{high})$, that is, if e_{j+1} is played, after the *high* report, continuing with "babbling" is non revision-proof. This implies that there exist $k > j + 1$ such that $e^{j+1}_k \in RP(G^{j+1}_{high})$, therefore $e^k_k \in RP((G^{j+1}_{high})^k_{high})$, and then $e^k_k \in RP(G^k_{high})$.

Consider $e_k \in EQ(G^0)$. There does not exist $t \geq k$ and a $t-$ *continuation* of G^0 and e_k, in which e_k is trumped (that is there does not exist $\tilde{G} \in \Gamma_t(L)$ and $\tilde{e} \in RP(\tilde{G})$ such that $G^0 \underset{e_k}{\to} \tilde{G}$ and $\tilde{e} \succ e_k|_{\tilde{G}}$). Moreover, if e_k is trumped at $t_1 < k$, it is trumped by an equilibrium that induces the partition of e_{k_1} for some $k_1 > k$ where the sender's information structure is given by L_{t_1}. If $t_1 > 0$, there exists $t_2 < t_1$ such that the equilibrium that induces the partition of e_{k_1} is trumped by an equilibrium that induces the partition of e_{k_2} for some $k_2 > k_1$ where the sender's information structure is given by L_{t_2}. Continue in the same manner and notice that $\{t_i\}$ is strictly increasing and $\{k_i\}$ is strictly decreasing. Thus, there must exist i^* such that $k_{i^*} > j$ and $e_{k_{i^*}} \in RP(G^0)$. A contradiction. □

We now make a few short remarks surrounding the above example. Firstly, consider the case where, instead of the infinite horizon, we had the same learning environment which terminates after $T \in \mathbb{N}$ periods. The equilibria take the same form. If this were the case, both the ex-ante efficient equilibrium and unique revision-proof equilibrium would reveal information only in the last period. Therefore, in the infinite horizon not only the revision-proof but also the ex ante efficient equilibrium fails to exist (as they can always jointly benefit by delaying the single round of informative communication ex ante). The impossibility of identifying whether an equilibrium is revision-proof or not in the infinite horizon, as illustrated in Example 2, is, however, not limited to situations where the ex-ante efficient equilibrium does not exist. To illustrate this in a simple way we build on Example 2 as follows.

Let us extend the state space to $\{[-\frac{2}{3}, 1]\}$ and add, *before* the learning in Example 2, another period of learning. Specifically, suppose the sender is initially uninformed, but now, in period "0" the sender learns the state perfectly if it is negative or learns that $\theta \in [0, 1]$ if it is non-negative—and from $t = 1$ onwards (conditional on $\theta \in [0, 1]$), the continuation is identical to that in Example 2.

Now, there is an equilibrium where the sender only reveals whether the state is negative or not. Moreover, this is the ex-ante efficient equilibrium, both in the finite and the infinite case. In the finite case, this is the unique revision-proof equilibrium. Nonetheless, in the infinite horizon setting, the same issue of Example 2 is inherited after revealing that $\theta \in [0, 1]$, the dynamically consistent partition does not exist.

References

Ales, L. and Sleet, C. (2014). Revision proofness. *Journal of Economic Theory*, 152:324–355.

Asheim, G. (1991). Individual and collective time consistency. *No 1991-69, Discussion Paper from Tilburg University, Center for Economic Research*.

Asheim, G. B. (1997). Individual and collective time-consistency. *The Review of Economic Studies*, 64(3):427–443.

Bernheim, B. D., Peleg, B., and Whinston, M. D. (1987). Coalition-proof nash equilibria i. concepts. *Journal of economic theory*, 42(1):1–12.

Bernheim, B. D. and Ray, D. (1989). Collective Dynamic Consistency in Repeated Games. *Games and Economic Behavior*, 1(4):295–326.

Bloch, F. and Dutta, B. (2009). Correlated equilibria, incomplete information and coalitional deviations. *Games and Economic Behavior*, 66(2):721–728.

Crawford, V. P. and Sobel, J. (1982). Strategic information transmission. *Econometrica: Journal of the Econometric Society*, pages 1431–1451.

de Clippel, G. (2007). The type-agent core for exchange economies with asymmetric information. *Journal of Economic Theory*, 135(1):144–158.

Dutta, B. and Vohra, R. (2005). Incomplete information, credibility and the core. *Mathematical Social Sciences*, 50(2):148–165.

Farrell, J. and Maskin, E. (1989). Renegotiation in Repeated Games. *Games and Economic Behavior*, 1(4):327–360.

Frug, A. (2016). Dynamic cheap talk with static informational control. *Economics Letters*, 143:118–120.

Frug, A. (2018). Strategic gradual learning and information transmission. *Journal of Economic Theory*, 177:594–615.

Frug, A. (2022). On strategic transmission of gradually arriving information. *Available at SSRN 4279000*.

Greenberg, J. (1990). *The theory of social situations: an alternative game-theoretic approach*. Cambridge University Press.

Hellwig, M. and Leininger, W. (1987). On the existence of subgame-perfect equilibrium in infinite-action games of perfect information. *Journal of Economic Theory*, 43(1):55–75.

Ivanov, M. (2015). Dynamic information revelation in cheap talk. *The BE Journal of Theoretical Economics*, 15(2):251–275.

Ivanov, M. (2016). Dynamic learning and strategic communication. *International Journal of Game Theory*, 45:627–653.

Lucas, W. F. (1969). The proof that a game may not have a solution. *Transactions of the American Mathematical Society*, 137:219–229.

Pollak, R. A. (1968). Consistent planning. *The Review of Economic Studies*, 35(2):201–208.

Richardson, M. (1953). Solutions of irreflexive relations. *Annals of Mathematics*, 58(3):573–590.

Salamanca, A. (2020). A generalization of the harsanyi ntu value to games with incomplete information. *International Journal of Game Theory*, 49(1):195–225.

Strotz, R. H. (1955). Myopia and inconsistency in dynamic utility maximization. *Review of Economic Studies*, 23(3):165–180.

Xue, L. (2000). Negotiation-proof nash equilibrium. *International Journal of Game Theory*, 29:339–357.